CSC 665: Midterm

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Please complete the following set of problems. You must do the exercises completely on your own (no collaboration allowed this time). The exam is due **on Oct 24, 12:30pm, on Gradescope**. You are free to cite existing theorems from the textbooks and course notes.

Problem 1

Define $\mathcal{H} = \left\{\text{sign}(p(x)) : p \text{ is a polynomial of } x \text{ of degree} \leq n\right\}$ (where $x \in \mathbb{R}$). Here $\text{sign}(z) = 2\mathbf{1}(z > 0) - 1$. What is the VC dimension of $\mathcal{H}$?

Problem 2

Suppose we have an algorithm $\mathcal{B}$ that learns hypothesis class $\mathcal{H}$ in the following sense. There exists a function $m(\epsilon)$, such that for any $\epsilon > 0$, suppose $\mathcal{B}$ draws $m \geq m(\epsilon)$ training examples from a distribution D realizable by $\mathcal{H}$, then with probability $\geq \frac{1}{2}$, $\mathcal{B}$ returns a classifier $\hat{h}$ with error at most ϵ on D.

Now, given $\mathcal{B}$ and the ability of drawing fresh training examples, how can you design an algorithm $\mathcal{A}$ that (ϵ, δ)-PAC learns $\mathcal{H}$ for any ϵ, δ? What is its sample complexity? (You may want to run $\mathcal{B}$ multiple times.)

Problem 3

Suppose $X_1, \ldots, X_n$ is a sequence of n iid random variables, and let $\sigma^2 = \text{var}(X_i)$ and $\mu = \mathbb{E}(X_i)$. Suppose $n = mk$ for some integer m and odd integer $k \geq 20 \ln \frac{1}{\delta}$. Denote by

$$\hat{\mu} = \text{median}(\hat{\mu}_1, \ldots, \hat{\mu}_k),$$

where $\hat{\mu}_i = \frac{1}{m} \sum_{j=(i-1)m+1}^{im} X_j$.

1. Show that for every j,
$$\mathbb{P}(|\hat{\mu}_j - \mu| \leq \frac{2\sigma}{\sqrt{m}}) \geq \frac{3}{4}.$$

2. Show that
$$\mathbb{P}(|\hat{\mu} - \mu| \leq \frac{2\sigma}{\sqrt{m}}) \geq 1 - \delta.$$